

# Analysis of Circular Stripline Resonators on Normally Biased Ferrite Substrates

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**Abstract**—Galerkin's method in the Hankel transform domain is used for determining the resonant frequencies of stripline circular resonators on normally biased ferrite substrates. The numerical results obtained show that the resonant frequencies of the resonators can be tuned by varying the magnitude of the applied bias magnetic field. However, there is a cutoff frequency band in which resonances are not allowed owing to the excitation of an infinite number of magnetostatic volume-wave modes along the ferrite substrates supporting the resonators.

**Index Terms**—Ferrites, microwave devices and components, resonators, stripline.

## I. INTRODUCTION

CIRCULAR stripline resonators manufactured on ferrite substrates are used in stripline junction circulators [1]. Also, these structures may find an application in stripline oscillators and filters as resonators with magnetic tuning capabilities since their resonant frequencies can be varied by adjusting the applied bias magnetic field, as it happens with microstrip resonators printed on ferrite substrates [2], [3]. In this letter, the authors make use of Galerkin's method in the Hankel transform domain [2] for obtaining the resonant frequencies of the modes of circular stripline resonators with a normally applied bias magnetic field. It is shown that for every value of bias magnetic field, there is a band of frequencies in which resonances cannot occur owing to the excitation of an infinite number of magnetostatic volume-wave modes along the ferrimagnetic slabs supporting the resonators [4].

## II. METHOD OF ANALYSIS

In this work, Galerkin's method in the Hankel transform domain [2] is used for computing the resonant frequencies of circular stripline resonators on magnetized ferrite substrates (Fig. 2 shows the side and top views of one resonator of this type). The metallizations of the resonators (ground planes and circular patch) are assumed to be infinitely thin and lossless. The permeability tensor of the ferrite substrates with respect to the coordinates of the drawings of Fig. 2 is assumed to be given by [3, eqs. (1)–(3b)]. For axial-symmetric modes (modes with fields showing revolution symmetry around the  $z$  axis of Fig. 2), the components of the surface current density

on the circular metallic patch are approximated by using the following basis functions:

$$j_{\rho,i}^0(\rho) = U_{2i-1}(\rho/a)\sqrt{1-(\rho/a)^2} \quad (i = 1, \dots, N) \quad (1)$$

$$j_{\phi,i}^0(\rho) = j \frac{T_{2i-2}(\rho/a)}{\sqrt{1-(\rho/a)^2}}(\rho/a) \quad (i = 1, \dots, N+1) \quad (2)$$

where  $U_{2i-1}(\bullet)$  and  $T_{2i-2}(\bullet)$  are Chebyshev polynomials of second and first kind, respectively.

For nonaxial symmetric modes with fields showing a dependence on the  $\phi$  coordinate of the type  $e^{jm\phi}$  ( $m = \dots -2, -1, 1, 2, \dots$ ) [2], the components of the surface current density on the patch are approximated by using the following basis functions:

$$j_{\rho,i}^m(\rho, \phi) = U_{2i-1}(\rho/a)\sqrt{1-(\rho/a)^2}(\rho/a)^{|m|-2}e^{jm\phi} \quad (m = \dots -2, -1, 1, 2, \dots; i = 1, \dots, N) \quad (3)$$

$$j_{\phi,i}^m(\rho, \phi) = j \frac{T_{2i-2}(\rho/a)}{\sqrt{1-(\rho/a)^2}}(\rho/a)^{|m|-1}e^{jm\phi} \quad (m = \dots -2, -1, 1, 2, \dots; i = 1, \dots, N+1). \quad (4)$$

The Hankel transforms of the basis functions of (1)–(4) can be obtained in closed form (in terms of spherical Bessel functions), which makes the use of these functions computationally efficient when applying Galerkin's method in the Hankel transform domain [2]. Also, these basis functions account for the edge singularities of the current density on the circular patch and they make it possible to reproduce the current density behavior around the center of the patch suggested by the cavity model [1]. Thanks to these two latter facts, the basis functions of (1)–(4) provide a very accurate approximation of the current density on the circular patch, which ensures a quick convergence of the resonant frequencies with respect to the number of basis functions when applying Galerkin's method. It should be pointed out that for the computation of the integrals involved in the application of Galerkin's method in the Hankel transform domain [2], it has been necessary to deform the integration path above the poles of the spectral Green's function [5]. (These poles correspond to the propagation constants of the modes that can propagate along the ferrite slab surrounding the resonator.)

## III. NUMERICAL RESULTS

In order to check the validity of our numerical results for the resonant frequencies of circular stripline resonators on ferrite substrates, in Fig. 1 the results obtained with our

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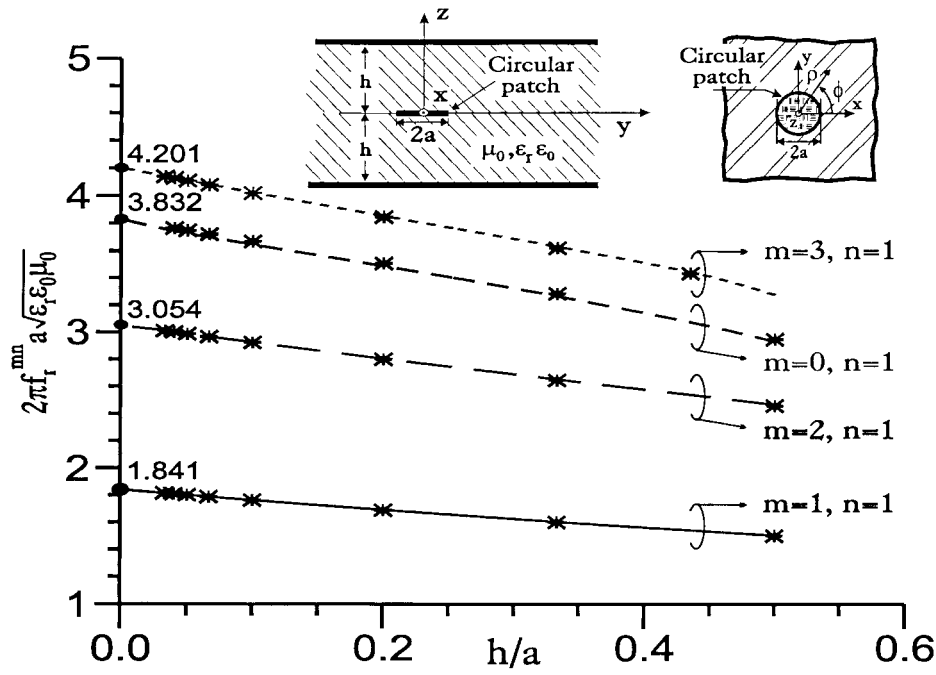


Fig. 1. Normalized resonant frequencies of the first resonant modes of circular stripline resonators on dielectric substrates. Galerkin's method results (solid lines) are compared with those obtained in [6] (\*).

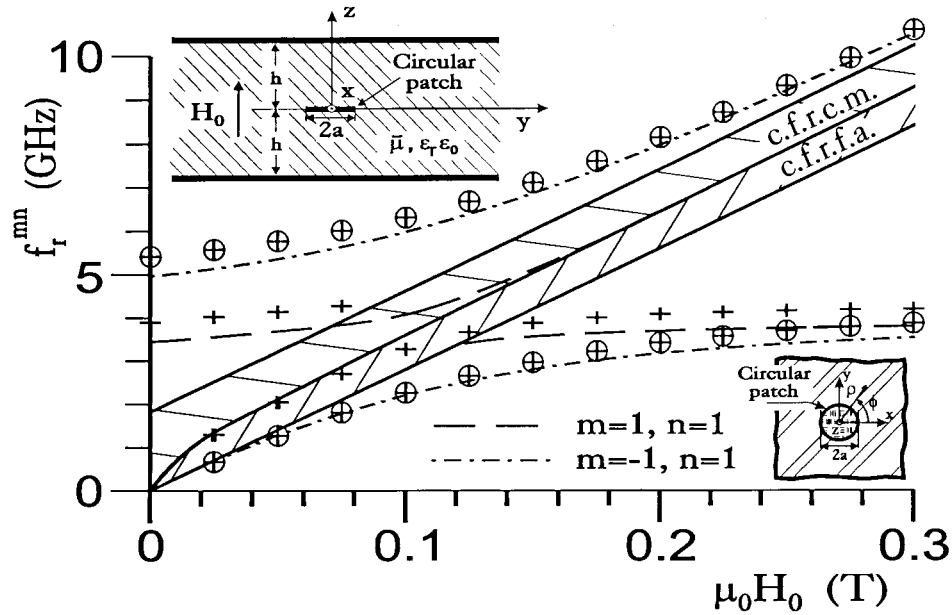


Fig. 2. Resonant frequencies of the first two resonant modes of a circular stripline resonator on a ferrite substrate. Galerkin's method results (dashed line, and dots and dashes) are compared with results obtained by means of the cavity model (+:  $m = 1, n = 1$ ;  $\oplus$ :  $m = -1, n = 1$ ). The shadowed region named "c.f.r.c.m." stands for the cutoff frequency region predicted by cavity model. The shadowed region named "c.f.r.f.a." stands for the cutoff frequency region predicted by full-wave analysis.  $\epsilon_r = 15$ ,  $\mu_0 M_s = 0.065$  T,  $h = 1.27$  mm,  $a = 5$  mm.

computer code for the resonant frequencies of a standard circular stripline resonator on a dielectric substrate are compared with the numerical results obtained in [6] via a mode-matching technique. Excellent agreement is found (differences lie within 2%) between the two sets of results. In Fig. 2 the resonant frequencies of the first two modes of a circular stripline resonator on a normally biased ferrite substrate are plotted against bias magnetic field. It can be noticed that the

aforementioned resonant frequencies can be tuned over a wide range by adjusting the value of the bias magnetic field, as it has been experimentally demonstrated for microstrip resonators on ferrite substrates [2]. In Fig. 2 the results obtained for the resonant frequencies via Galerkin's method are compared with those obtained by means of the cavity model [1] (in most cases, differences between these two sets of results are roughly around 10%). It can be seen that for every value of  $H_0$ ,

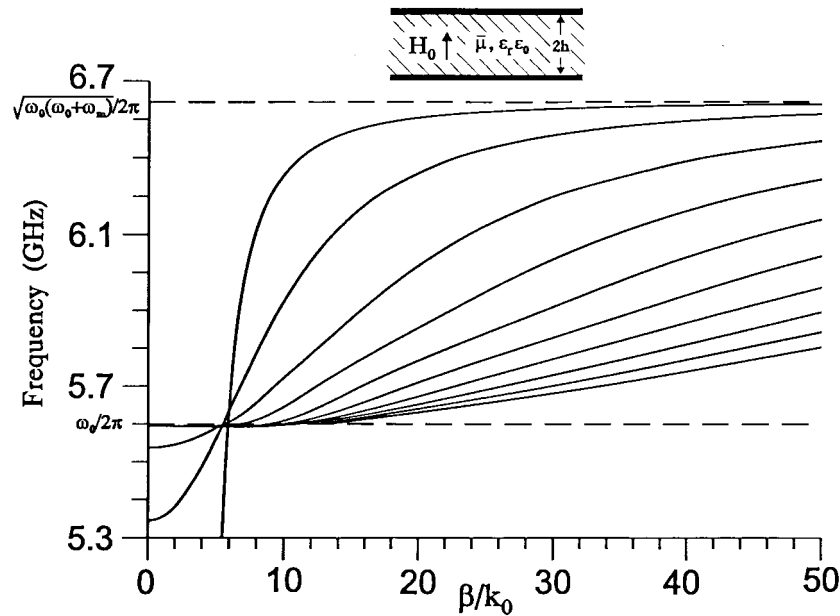


Fig. 3. Normalized phase constants (solid lines) of the magnetostatic volume-wave modes propagating along the ferrimagnetic slab supporting the circular stripline resonator of Fig. 2 when  $\mu_0 H_0 = 0.2$  T,  $\epsilon_r = 15$ ,  $\mu_0 M_s = 0.065$  T,  $h = 1.27$  mm.

there is a cutoff frequency region in which resonances are not allowed. The cavity model predicts the existence of this cutoff frequency region [3] in the interval  $\sqrt{\omega_0(\omega_0 + \omega_m)}/2\pi < f < (\omega_0 + \omega_m)/2\pi$  ( $\omega_0$  and  $\omega_m$  being defined in [3, eqs. (3a) and (3b)]). However, a rigorous full-wave analysis via Galerkin's method reveals that the cutoff frequency interval is  $\omega_0/2\pi < f < \sqrt{\omega_0(\omega_0 + \omega_m)}/2\pi$ . It has been found that the resonator cannot operate at these frequencies owing to the excitation of an infinite number of magnetostatic volume-wave modes along the ferrimagnetic slab supporting the resonator (see Fig. 3), which is something that cannot be explained by means of the cavity model.

Finally, in Fig. 3 results are presented for the normalized phase constants of magnetostatic volume-wave modes propagating along the ferrimagnetic slab used as a substrate for the circular stripline resonator of Fig. 2. These results have been generated by means of a rigorous full-wave analysis [7]. As mentioned above, Fig. 3 shows that an infinite number of magnetostatic volume-wave modes are excited inside the ferrite slab in the frequency interval  $[\omega_0/2\pi < f < \sqrt{\omega_0(\omega_0 + \omega_m)}/2\pi]$ . Also, a finite number of these magnetostatic volume-wave modes may propagate below the resonant frequency of the ferrite,  $f_0 = \omega_0/2\pi$  (as it happens in [7]). However, these latter magnetostatic modes do not prevent resonances from occurring when  $f < f_0 = \omega_0/2\pi$  as shown in Fig. 2. Although ferrite losses have been neglected when computing the results of Figs. 2 and 3, the authors have numerically checked that when these losses are accounted for (i.e., when  $\Delta H \neq 0$  as described in [1]), the results shown in Figs. 2 and 3 do not substantially change (the main effects of ferrite losses are to lower the quality factors of the resonant

modes of the stripline resonator and to set attenuation constants for volume-wave magnetostatic modes).

#### IV. CONCLUSIONS

Galerkin method in the Hankel transform domain is applied to the determination of the resonant frequencies of the modes of circular stripline resonators on normally biased ferrite substrates. It is shown that these resonant frequencies can be varied over a wide range by adjusting the applied bias magnetic field. Also, it is shown that there is a cutoff frequency region in which the resonator cannot operate owing to the excitation of magnetostatic volume-wave modes along the ferrimagnetic substrate. The cavity model has proved to be both inaccurate for the computation of the resonant frequencies and unsound for the prediction of the cutoff frequency interval.

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